Acta Technica 62 No. 3B/2017, 173–180

Modeling and control of embedded underwater robot based on backstepping adaptive algorithm[1]

Wei Chen[2], Chenyu Peng[3,5], Siyu Chen[4], Xiuping Wen[2], Jun Zhang[2], Hao Wang[4]

Abstract. The attitude and depth stability of underwater robots plays a decisive role in its underwater operating accuracy. However, underwater environment is convoluted due to multiple interference factors such as flow influence, wall effect and among others. It is necessary to design a motion control system, of which the control functions and accuracy meet the requirements, to ensure a high stability of attitude in actual working process. Based on the mechanical structures of existing underwater robots, this article puts forward the R&D of a motion control system based on embedded underwater robot; it describes in details the quaternion-based Strap-down Inertial Navigation System (SINS) and accurately obtains the attitude model through simulating power system layout of four-rotor aircraft; in addition, it innovatively adopts the backstepping adaptive algorithm in underwater robot motion control to improve the attitude control accuracy. The results of underwater robot simulation have a good consistency with the experimental data under various flow conditions.

Key words. Underwater search & rescue robot, quaternion, strap-down inertial navigation system (SINS), backstepping algorithm,.

[1]The authors are highly thankful for the financial support of Major Projects of Innovation Fund of Nanjing Institute of Technology (No. CKJA201606), Practice Innovation Training Projects for-College Students in Jiangsu Province in 2015 (No. 201511276001Z), Support Projects of 2016 Challenge Cup Competition from University Students Science and Technology Innovation Fund of Nanjing Institute of Technology (No. TP20160009 & No. TZ20160028), Practice Innovation Training Projects for College Students in Jiangsu Province in 2015 (No.201511276001Z), Introduce Talents Start-Up Funding Scientific Research Projects of Nanjing Institute of Technology (YKJ201336 & No.YKJ201337) and Youth Funded Projects of Nan Jing Institute of Technology (QKJB201410).

[2]Industrial Center, Nanjing Institute of Technology, Jiangning District, Nanjing, 211167, China

[3]Viterbi School of Engineering, University of Southern California, Los Angeles, CA 90089, US

[4]School of Electric Power Engineering, Nanjing Institute of Technology, Jiangning District, Nanjing, 211167, China

[5]Corresponding author

1. Introduction

The total surface area of the Earth is 510 million square kilometers [1], of which ocean accounts for about 70 percent. Ocean is rich in marine living resources, mineral resources and other resources, and it is one of the four strategic spaces for human survival and development. In this context, the development of underwater detection equipment has become an important guarantee for the marine maintenance and full utilization of marine resources; the development of intelligent and multi-function unmanned underwater vehicles (UUV) has become the wave of the future.

2. Establishment of the dynamic model

2.1. Dynamic system model.

The dynamical system of underwater search & rescue robot mainly includes the motor drive, propeller and speed measurement [2]. The model is as follows:

$$J_{\mathrm{total}}\dot{\omega}_{\mathrm{motor}} = K_{\mathrm{M}}I - M_{\mathrm{drag}}\,, \tag{1}$$

$$V = IR + K_{\mathrm{E}}\omega_{\mathrm{motor}}\,, \tag{2}$$

$$J_{\mathrm{total}} = J_{\mathrm{R}} + J_{\mathrm{rotor}\to\mathrm{motor}}\,, \tag{3}$$

$$M_{\mathrm{drag}} = \frac{M_{\mathrm{rotor}}\theta_2}{\mu\theta_1} = \frac{M_{\mathrm{rotor}}}{\eta\tau}\,, \tag{4}$$

$$M_{\mathrm{drag}} = \frac{k_{\mathrm{d}}\omega_{\mathrm{rotor}}^2}{\eta\tau} = \frac{k_{\mathrm{d}}\omega_{\mathrm{motor}}^2}{\eta\tau}\,, \tag{5}$$

$$J_{\mathrm{total}} = K_{\mathrm{M}}\left(\frac{V - K_{\mathrm{E}}\omega_{\mathrm{motor}}}{R}\right) - \frac{k_{\mathrm{d}\omega}\omega_{\mathrm{motor}}^2}{\eta\tau^3}\,, \tag{6}$$

$$\dot{\omega}_{\mathrm{rotor}} = -\frac{k_{\mathrm{d}}}{J_{\mathrm{total}}\eta\tau^2}\omega_{\mathrm{rotor}}^2 - \frac{K_{\mathrm{M}}K_{\mathrm{E}}}{J_{\mathrm{total}}R}\omega_{\mathrm{rotor}} + \frac{K_{\mathrm{M}}}{J_{\mathrm{total}}R\tau}V\,. \tag{7}$$

Formulae (1) and (2) are the general equations of the model. Here, J_{total} is the total moment of inertia, ω_{motor} is the motor speed, ω_{rotor} is the rotor speed K_{M} is the motor torque constant, M_{drag} is the motor load torque, V is the motor input voltage, I is the armature current, R is the rcuit resistance, θ_1 and θ_2 are the motor rotational angles and K_E is the back electromotive force constant [3].

Formula (3) is the synthesis equation of total moment of inertia. Symbol J_{R} is the motor rotor inertia, and $J_{\mathrm{rotor}\to\mathrm{motor}}$ is the moment of inertia of other parts [4].

Formula (4) can be obtained through energy conservation equation $\eta M_{\mathrm{drag}}\theta_1 = M_{\mathrm{drag}}\theta_2$.

Formula (5) is obtained through the simplification of formula (4) by assuming the scale factor being k_{d}.

Formulae (6) and (7) represent the final dynamical equations of the dynamical system.

2.2. Model establishment

Define F_X, F_Y, F_Z as the components of force $\boldsymbol{F}$ in three axis of coordinate system. Let p, q and r be the components of angular velocity $\boldsymbol{\omega}$ of underwater vehicle. Using Newton's Second Law of Motion, the dynamical equations of underwater vehicle can be separately expressed in vector forms [5]

$$\boldsymbol{F} = m\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}, \quad \boldsymbol{M} = \frac{\mathrm{d}\boldsymbol{H}}{\mathrm{d}t}, \tag{8}$$

where $\boldsymbol{F}$ is the sum of external forces acting on underwater robot, m is the mass of underwater robot, $\boldsymbol{V}$ is its velocity of underwater vehicle, $\boldsymbol{M}$ is the sum of torques acting on the underwater vehicle and $\boldsymbol{H}$ is the absolute angular momentum of the underwater robot relative to the ground coordinate system and symbol $\boldsymbol{G}$ denotes the gravity. There also holds

$$\boldsymbol{G} = m\boldsymbol{g}, \quad D_i = \rho C_{\mathrm{d}}\omega_{\mathrm{i}}^2/2 = k_{\mathrm{d}}\omega_{\mathrm{i}}^2, \quad T_i = \rho C_{\mathrm{t}}\omega_{\mathrm{i}}^2/2 = k_t\omega_{\mathrm{i}}^2, \tag{9}$$

where ϱ is the density of water, C_{d} and C_{t} are the drag coefficients, T_i is the lifting force of propeller and ω_{i} is the angular velocity.

According to the stress analysis, Newton's Second Law of Motion and underwater vehicle dynamical equation, the component equations of the movement can be obtained and expressed as follows:

$$\begin{aligned} \ddot{x} &= (F_x - K_1\dot{x})/m = \left(k_t\sum_{i=1}^{4}\omega_i^2(\sin\psi\sin\theta\sin\phi + \sin\psi\sin\phi) - K_1\dot{x}\right)/m, \\ \ddot{y} &= (F_y - K_2\dot{y})/m = \left(k_t\sum_{i=1}^{4}\omega_i^2(\sin\psi\sin\phi\cos\phi - \cos\psi\sin\phi) - K_1\dot{x}\right)/m, \\ \ddot{z} &= (F_z - K_3\dot{z} - mg)/m = \left(k_t\sum_{i=1}^{4}\omega_i^2(\cos\phi\cos\phi) - K_1\dot{x}\right)/m - g. \end{aligned} \tag{10}$$

Here, K_1, K_2 and K_3 are the scaling factors.

According to the relationship between the Euler angle and angular velocity of underwater vehicle, it can be obtained

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}\frac{p\cos\theta + q\sin\phi\sin\theta + r\cos\phi\sin\theta}{\cos\theta}\\ q\cos\phi + r\sin\phi\\ \frac{q\sin\phi + r\cos\phi}{\cos\theta}\end{bmatrix}, \tag{11}$$

3. Aircraft attitude detection and control principle

As shown in the Fig. 1, we define coordinate system A by $\mathrm{O}, X_{\mathrm{A}}, Y_{\mathrm{A}}, Z_{\mathrm{A}}$, and coordinate system B by $\mathrm{O}, X_{\mathrm{B}}, Y_{\mathrm{B}}, Z_{\mathrm{B}}$. Assume that ${}^{\mathrm{A}}\gamma = (r_x, r_y, r_z)$ is an arbitrary vector in A, and we will express an arbitrary angle θ from B relative to A by its rotation around ${}^{\mathrm{A}}\gamma$. We usually express this angle by quaternion ${}^{\mathrm{A}}_{\mathrm{B}}q$ [6] defined as

$$ {}^{\mathrm{A}}_{\mathrm{B}}q = [q0\, q1\, q2\, q3] = \left[\cos\frac{\theta}{2} \quad -\gamma_x \sin\frac{\theta}{2} \quad -\gamma_y \sin\frac{\theta}{2} \quad -\gamma_z \sin\frac{\theta}{2}\right], \tag{12} $$

where $-\gamma_x$ is the component of vector ${}^{A}\gamma$ in axis X of the coordinate system A, $-\gamma_y$ and $-\gamma_z$ are components in axis Y and axis Z, respectively.

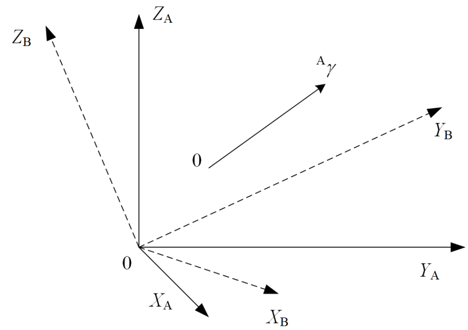

Fig. 1. Coordinate system A revolves around ${}^{\mathrm{A}}\gamma$

Set that at a moment, moving coordinate system O_{XYZ} rotates relatively to fixed coordinate system $\mathrm{O}_{X_t Y_t Z_t}$ by Q_1, i.e.

$$ R_t(t) = Q_1 R(t) Q_1^{-1}. \tag{13} $$

here $Q_1()Q_1^{-1}$ is the rotation operator and Q_1 is the rotation quaternion [7].

At the moment $t + \Delta t$ we have (see Fig. 3)

$$ R_t(t+\Delta t) = Q_2 R(t+\Delta t) Q_2^{-1}. \tag{14} $$

Then it can be obtained that at the moment of change t to $t + \Delta t$, the position change of moving coordinate can be expressed as $Q_1^{-1}Q_2$, and their relationship is as shown in Fig. 3. As Δt is very small, the angular velocity of moving coordinate $\overline{\omega}$ is constant, so that the angular displacement of moving coordinate is

$$ \Delta\theta = |\overline{\omega}|\Delta t. \tag{15} $$

In formula (15), $|\overline{\omega}|$ expresses the module of $\overline{\omega}$ that determines the direction of $\Delta\theta$. Set unit vector $\xi = \frac{\overline{\omega}}{|\overline{\omega}|}$, then it can be obtained that

$$ Q_1^{-1}Q_2 = \cos\frac{|\overline{\omega}|\,\Delta t}{2} + \xi \sin\frac{|\overline{\omega}|\,\Delta t}{2} \tag{16} $$

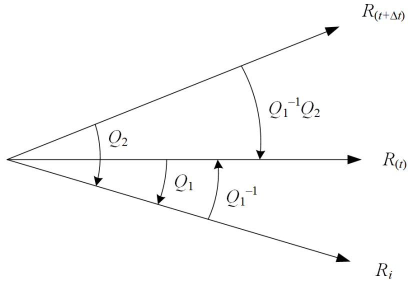

Fig. 2. Rotation quaternion change diagram

and hence,

$$Q_2 = Q1\left(\cos\frac{|\varpi|\,\Delta t}{2} + \xi\sin\frac{|\varpi|\,\Delta t}{2}\right). \tag{17}$$

Therefore, the derivative of $Q(t)$ with respect to time $\overset{\bullet}{Q}(t)$ can be expressed as follows

$$\overset{\bullet}{Q}(t) = \lim_{\Delta t\to 0}\frac{Q_2 - Q_1}{\Delta t} = \lim_{\Delta t\to 0}\frac{Q_1}{\Delta t}\left(\cos\frac{|\varpi|\,\Delta t}{2} - 1 + \xi\sin\frac{|\varpi|\,\Delta t}{2}\right). \tag{18}$$

As ϖ only has the vector part, the rotation quaternion differential equation is

$$\overset{\bullet}{Q}(t) = \frac{1}{2}Q\xi\,|\varpi| = \frac{1}{2}Q\varpi = \frac{1}{2}Q\omega\,, \tag{19}$$

so that the correction model of quaternion may be expressed as

$$\overset{\bullet}{Q} = \begin{bmatrix}\overset{\bullet}{q}_0\\ \overset{\bullet}{q}_1\\ \overset{\bullet}{q}_2\\ \overset{\bullet}{q}_3\end{bmatrix} = \frac{1}{2}\omega^{\mathrm{b}}_{\mathrm{nb}}Q = \begin{bmatrix}0 & -\omega^{\mathrm{b}}_{\mathrm{nb}x} & -\omega^{\mathrm{b}}_{\mathrm{nb}y} & -\omega^{\mathrm{b}}_{\mathrm{nb}z}\\ -\omega^{\mathrm{b}}_{\mathrm{nb}x} & 0 & -\omega^{\mathrm{b}}_{\mathrm{nb}z} & -\omega^{\mathrm{b}}_{\mathrm{nb}y}\\ -\omega^{\mathrm{b}}_{\mathrm{nb}y} & -\omega^{\mathrm{b}}_{\mathrm{nb}z} & 0 & -\omega \mathrm{nb}x^{\mathrm{b}}\\ -\omega^{\mathrm{b}}_{\mathrm{nb}z} & -\omega^{\mathrm{b}}_{\mathrm{nb}y} & -\omega^{\mathrm{b}}_{\mathrm{nb}x} & 0\end{bmatrix}\cdot\begin{bmatrix}q_0\\ q_1\\ q_2\\ q_3\end{bmatrix} \tag{20}$$

The next step is calculation of the strap-down matrix T. By using the four units q_0, q_1, q_2 and q_3 in the quaternion, the strap-down matrix can be obtained in the form

$$T = \begin{bmatrix}q_0^2+q_1^2-q_2^2-q_3^2 & 2\left(q_1q_2-q_0q_3\right) & 2\left(q_1q_3+q_0q_2\right)\\ 2\left(q_1q_2+q_0q_3\right) & q_0^2-q_1^2+q_2^2-q_3^2 & 2\left(q_2q_3-q_0q_1\right)\\ 2\left(q_1q_3+q_0q_2\right) & 2\left(q_2q_3+q_0q_1\right) & q_0^2+q_1^2-q_2^2+q_3^2\end{bmatrix} \tag{21}$$

The attitude matrix T can also be expressed by the relationship of roll angle ϕ, pitch angle θ and drift angle ψ, as follows:

$$T = \begin{bmatrix}\cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi & -\cos\theta\sin\psi & \sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi\\ \cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi & \cos\theta\cos\psi & \sin\phi\sin\psi - \cos\phi\sin\theta\cos\psi\\ -\sin\phi\cos\theta & \sin\theta & \cos\phi\cos\theta\end{bmatrix} =$$

$$= \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} . \tag{22}$$

Then, the principal values of pitch angle θ, roll angle ϕ and drift angle ψ are

$$\theta_{\mathrm{p}} = \sin^{-1}(T_{32}) \, , \ \phi_{\mathrm{p}} = \tan^{-1}(\frac{-T_{31}}{-T_{33}}) \, , \ \psi_{\mathrm{p}} = \tan^{-1}(\frac{-T_{12}}{-T_{22}}) \, . \tag{23}$$

According to the range of each angle θ, ϕ and ψ, the correct value of attitude angle can be calculated as

$$\theta = \theta_{\mathrm{p}} \, ,$$

$$\phi = \begin{cases} \phi_{\mathrm{p}} & (T_{33} > 0) \\ \phi_{\mathrm{p}} + 180^\circ & (T_{33} < 0, \quad \phi_{\mathrm{p}} < 0) \\ \phi_{\mathrm{p}} + 180^\circ & (T_{33} < 0, \quad \phi_{\mathrm{p}} > 0) \ , \end{cases}$$

$$\psi = \begin{cases} \psi_{\mathrm{p}} & (T_{22} > 0, \quad \psi_{\mathrm{p}} < 0) \\ \psi_{\mathrm{p}} + 360^\circ & (T_{22} < 0, \quad \psi_{\mathrm{p}} < 0) \\ \psi_{\mathrm{p}} + 180^\circ & (T_{22} < 0) \ . \end{cases} \tag{24}$$

First, we give the state space equation of the filter in the form

$$\begin{bmatrix} x_r \\ \bullet \\ x_r \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \cdot \begin{bmatrix} x_r \\ \bullet \\ x_r \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} \cdot x_r^0 \, , \tag{25}$$

where ξ is the damping ratio of the second-order filter, and ω_n is the natural frequency. A typical second-order low-pass filter is depicted in Fig. 4.

The transfer function from the input x_r^0 to the outputx_r is

$$\frac{x_r(s)}{x_r^0(s)} = G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \, . \tag{26}$$

It can be seen that an increase of ω_n can reduce error $\left|x_r^0(t) - x_r(t)\right|$, but if ω_n is too high, it will bring high-frequency measurement noise into the system.

4. Conclusion

This article analyzes the overall idea of underwater robot design and focuses on the layout of propellers; then it briefly analyzes the forces acting on underwater robots in the process of underwater movement and introduces two coordinate systems to describe the movements of underwater robots; in addition, it conducts a stress analysis on the space motion of underwater robot, and builds a dynamical model based on the analysis; at last, this article establishes the simulation model of underwater robots based on the previous study. It discusses the attitude detection and control principle of underwater robot, and introduce in detail the quaternion-based strap-down attitude description and obtains an accurate attitude model; then, it simulates the dynamical system layout of four-rotor aircraft, and innovatively adopts the control method of Backstepping into the motion control of underwater robot; in addition, it conducts control simulation experiments to prove the correctness and effectiveness of this method.

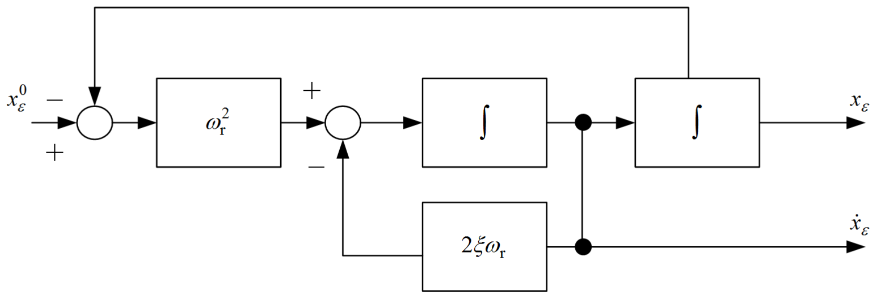

Fig. 3. Structure of the second-order low-pass filter

References

[1] J. Coffey: *Surface area of the earth.* Astronomy, Universe Today, 2009. [Online], `http://www.universetoday.com/25756/surface-area-of-the-earth/`. Accessed: 11 Apr. 2016.

[2] D. R. Yoerger, J. G. Cooke, J. J. E. Slotine: *The influence of thruster dynamics on underwater vehicle behavior and their incorporation into control system design.* IEEE Journal of Oceanic Engineering *15* (1990), No. 3, 167–178.

[3] N. E. Leonard, J. G. Graver: *Model based feedback control of autonomous underwater gliders.* IEEE Journal of Oceanic Engineering *26* (2001), No. 4, 633–645.

[4] A. Thondiyath: *Autonomy for robots: Design and developmental challenges (Keynote address).* Procedia Technology *23* (2016), No. 4, 4–6.

[5] J. Yuh: *Modeling and control of underwater robotic vehicles.* IEEE Transactions on Systems, Man, and Cybernetics *20* (1990), No. 6, 1475–1483.

[6] Y. González, G. Oliver: *Underwater scan matching using a mechanical scanned imaging sonar.* IFAC Proceedings Volumes *43* (2010), No. 16, 377–382.

[7] J. Yuh: *Design and control of autonomous underwater robots: A survey.* Autonomous Robots *8* (2000), No. 1, 7–24.

Received November 16, 2016